Kapittel I - Vektorer: Oppgave 197 og 198

197

a)
Kuleflate med sentrum i Q:
$(x^2 - 2x + 1^2) + (y^2 + 6y + 3^2) + (z^2 - 6z + 3^2) = 30 + 1^2 + 3^2 + 3^2 \Leftrightarrow$
$(x-1)^2 + (y+3)^2 + (z-3)^2 = 7^2$
): $\quad Q = (1, -3, 3) \qquad R_q = 7$
Kuleflate med sentrum i P:
$(x^2 - 10x + 5^2) + (y^2 - 18y + 9^2) + z^2 = -6 + 5^2 + 9^2 \Leftrightarrow$
$(x-5)^2 + (y-9)^2 + z^2 = 10^2$
): $\quad P = (5, 9, 0) \qquad R_p = 10$

b)
$\overrightarrow{PQ} = [-4, -12, 3] \qquad \Rightarrow \qquad \left|\overrightarrow{PQ}\right| = \sqrt{4^2 + 12^2 + 3^2} = 13$
$\alpha = \triangle QPR$
Cos-setningen gir:
$PQ^2 + PR^2 - 2 \cdot PQ \cdot PR \cos\alpha = RQ^2 \Rightarrow \cos\alpha = \frac{PQ^2 + PR^2 - RQ^2}{2 \cdot PQ \cdot PR} = \frac{13^2 + 10^2 - 7^2}{2 \cdot 13 \cdot 10} = \frac{11}{13}$
$PS = PR\cos\alpha = 10 \cdot \frac{11}{13} = \frac{110}{13}$
c)
Romfiguren er avgrenset av to kulesegmenter; formel: $O = 2\pi rh$ (Se side 66!)

$h_1 = PR - PS = 10 - \frac{110}{13} = \frac{20}{13} = 1.538\,461\,538\,461\,54$
$h_2 = RQ - SQ = RQ - (PQ - PS) = 7 - (13 - \frac{110}{13}) = \frac{32}{13}$

$O = 2\pi R_p h_1 + 2\pi R_q h_2 = 2\pi(R_p h_1 + R_q h_2) = 2\pi(10 \cdot \frac{20}{13} + 7 \cdot \frac{32}{13}) = \frac{848}{13}\pi \approx 204.9$

198

a) Figur...

b)
Planene som møtes i Origo ligger i henholdsvis $xy-$, $yz-$ og $zx-$planet, og da vi har et rettvinklet koordinatsystem, står planene normalt på hverandre.

c)
$\overrightarrow{OA} = [8, 0, 0], \qquad \left|\overrightarrow{OA}\right| = 8$
$\overrightarrow{OB} = [0, 5, 0], \qquad \left|\overrightarrow{OB}\right| = 5$
$\overrightarrow{OC} = [0, 0, 6], \qquad \left|\overrightarrow{OC}\right| = 6$
$A_1 = A_{OAB} = \frac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right| = \frac{1}{2}\sqrt{\left|\overrightarrow{OA}\right|^2 \left|\overrightarrow{OC}\right|^2 - \overline{(\overrightarrow{OA} \cdot \overrightarrow{OB})^2}} = \frac{1}{2}\sqrt{8^2 5^2 - 0^2} = 20$
(Eventuelt: $\frac{1}{2}|[8, 0, 0] \times [0, 5, 0]| = \frac{1}{2}|[0, 0, 40]| = \frac{1}{2}40 = 20$)
Tilsvarende:
$A_2 = A_{OBC} = \frac{1}{2}\left|\overrightarrow{OB} \times \overrightarrow{OC}\right| = \frac{1}{2}\sqrt{5^2 6^2 - 0^2} = 15$
$A_3 = A_{OCA} = \frac{1}{2}\left|\overrightarrow{OC} \times \overrightarrow{OA}\right| = \frac{1}{2}\sqrt{6^2 8^2 - 0^2} = 24$

d)
$\overrightarrow{AB} = [-8, 5, 0], \qquad \left|\overrightarrow{AB}\right| = \sqrt{8^2 + 5^2 + 0^2} = \sqrt{89}$
$\overrightarrow{AC} = [-8, 0, 6], \qquad \left|\overrightarrow{AC}\right| = \sqrt{8^2 + 0^2 + 6^2} = 10$
$\overrightarrow{AB} \cdot \overrightarrow{AC} = 64$
$A_4 = A_{ABC} = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{1}{2}\sqrt{\left|\overrightarrow{AB}\right|^2\left|\overrightarrow{AC}\right|^2 - (\overrightarrow{AB} \cdot \overrightarrow{AC})^2} = \frac{1}{2}\sqrt{89 \cdot 100 - 64^2} =$
$\sqrt{1201} \approx 34.7$

$VS = A_1^2 + A_2^2 + A_3^2 = 20^2 + 15^2 + 24^2 = 1201$
$HS = A_4^2 = 1201$
QED

e)
Tilsvarende c) og d):
$\overrightarrow{OA} = [a, 0, 0], \qquad \left|\overrightarrow{OA}\right| = a$
$\overrightarrow{OB} = [0, b, 0], \qquad \left|\overrightarrow{OB}\right| = b$
$\overrightarrow{OC} = [0, 0, c], \qquad \left|\overrightarrow{OC}\right| = c$
$A_1 = A_{OAB} = \frac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right| = \frac{1}{2}\sqrt{\left|\overrightarrow{OA}\right|^2\left|\overrightarrow{OC}\right|^2 - (\overrightarrow{OA} \cdot \overrightarrow{OB})^2} = \frac{1}{2}\sqrt{a^2b^2 - 0^2} = \frac{ab}{2}$
Tilsvarende:
$A_2 = A_{OBC} = \frac{1}{2}\left|\overrightarrow{OB} \times \overrightarrow{OC}\right| = \frac{1}{2}\sqrt{b^2c^2 - 0^2} = \frac{bc}{2}$
$A_3 = A_{OCA} = \frac{1}{2}\left|\overrightarrow{OC} \times \overrightarrow{OA}\right| = \frac{1}{2}\sqrt{c^2a^2 - 0^2} = \frac{ca}{2}$

d)
$\overrightarrow{AB} = [-a, b, 0], \qquad \left|\overrightarrow{AB}\right| = \sqrt{a^2 + b^2 + 0^2} = \sqrt{a^2 + b^2}$
$\overrightarrow{AC} = [-a, 0, c], \qquad \left|\overrightarrow{AC}\right| = \sqrt{a^2 + 0^2 + c^2} = \sqrt{a^2 + c^2}$
$\overrightarrow{AB} \cdot \overrightarrow{AC} = a^2$
$A_4 = A_{ABC} = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{1}{2}\sqrt{\left|\overrightarrow{AB}\right|^2\left|\overrightarrow{AC}\right|^2 - (\overrightarrow{AB} \cdot \overrightarrow{AC})^2} = \frac{1}{2}\sqrt{(a^2 + b^2)(a^2 + c^2) - (a^2)^2} =$
$\frac{1}{2}\sqrt{a^2b^2 + a^2c^2 + b^2c^2}$

$VS = A_1^2 + A_2^2 + A_3^2 = \frac{a^2b^2}{4} + \frac{b^2c^2}{4} + \frac{c^2a^2}{4} = \frac{a^2b^2+b^2c^2+c^2a^2}{4}$
$HS = A_4^2 = \frac{a^2b^2+a^2c^2+b^2c^2}{4}$
QED